AN EOQ MODEL WITH PRICE AND TIME DEPENDENT DEMAND UNDER THE INFLUENCE OF COMPLEMENT AND SUBSTITUTE PRODUCT'S SELLING PRICES

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ABSTRACT

Demand for a product is affected by its own selling price. In many situations it has also been affected by its Complement's as well as substitute's selling prices. This effect can be observed in many retailed items all over the markets. In this paper, we developed finite replenishment inventory model while considering that demand is sensitive to changes in time, its selling price and prices of complementary and substitute good's. Numerical example and sensitivity analysis have been performed for demand equation parameters.
Keywords: *EOQ, inventory, selling price, substitute good, complement good*

1. INTRODUCTION

Traditional inventory models assume that the demand for item is constant along the cycle. This restrictive assumption posits that demand is exogenous and no outside force can alter the quantity demanded. However it has been widely known that demand for a product is negatively affected from its selling price. Relaxing the assumption, Whitin (1955) was the first to integrate price within inventory models, important step in integrating marketing and purchasing decisions rather than treating them independently. Since then many authors investigated this relationship. However the economic theory indicates that in many situations demand for a product not only affected by its own price but also its substitutes as well as complement products prices. Consequently, many retailers and producers can not set selling prices independently without considering the substitute product's as well as rival product's prices. In this paper we develop an EOQ model where demand depends not only on its own price but also on complementary and substitute products' selling prices as well as time. The model allows calculation of optimal selling price levels which maximize total profit level in each cycle. Numerical example and sensitivity analysis on demand parameters has been performed.

2. EFFECTS OF SUBSTITUTE AND COMPLEMENTARY PRODUCTS' PRICES ON DEMAND

The law of demand suggests that demand for a product is adversely affected by its own selling price. If the product's selling price falls demand rises or the selling price rises demand falls. Moreover, demand for one good also depends on the price of other related goods. The standard economic textbooks indicate that related products' for a product include complement as well as substitute products. Two goods are complements if demand for one also requires the other. For instance anyone using a standard automobile should also consume gasoline as complementary product. The effects of these complementary goods on many retailed items can be observed throughout the markets. Additional examples of complements are hamburgers and hamburger buns, printers and printer ink replacements and cameras and picture processing. Demand for a product rises (falls) when the price of its complement falls (rises). Consequently, likewise a product's own selling price, a change in a Complement product's selling prices could alter quantity demanded for another good through cross-price effects. Indeed, the degree of cross-price effect depends on the degree of complementariness. Two goods are substitutes (or rival) if one can be used in place of the other one. Many products that are on the market today have substitutes. For example, bread and crackers, stocks and bonds, two different brands of soft drinks or water, domestic and foreign cars, oats and corns, natural gas and electricity or two different brands of toothpaste are substitutes.

Substitutes are the products if the demand for one rises when the price of the other rises or if the demand for one falls when the price of the other falls. The change in a substitute product's selling price could alter quantity demanded for another good through cross-price effects. The degree of cross-price effects depends on the degree of rivalry. When customers evaluate two substitute products for purchase, they will pay attention to characteristics of both products, including its prices, and make purchasing decisions. For that reason, producer could determine the optimal pricing and production decisions for a product while considering its substitute's price level in order to attain an optimal profit level.

3. INCORPORATION OF PRICING INTO THE EOQ/EPQ LITERATURE

Traditionally inventory models assume that the demand of an item is exogenously determined to the model. Whitin (1955) first incorporated the effects of price on demand within an inventory model, important step in integrating marketing and purchasing decisions rather than treating them independently. Since then many authors investigated this relationship. Keunreuther and Richard (1971), Cohen (1977), Rajan et al. (1992) consider problems with decaying inventories. Thomas (1970) and Wagner and Whitin (1958) analyze discrete-time, multi period models with concave costs. Abad (1988), Burwell et al. (1991) and Lee (1993) study a retailer's order quantity and procurement decisions when the supplier offers quantity discounts. Kim and Lee (1998) studied a model permit the maximum permissible demand rate to be expanded at a cost. Urban (1992) investigated a finite replenishment inventory model in which the demand of an item is a deterministic function of price and advertising expenditures. Kunreuther and Schrage (1973) and Gilbert (1999) study a model in which a constant price is to be maintained throughout the horizon. Jorgensen et al. (1999) have developed a model that considers learning effects. Also, several authors such as Ladany and Sternlieb (1974), Subramanyam and Kumaraswamy (1981), Brahmbahatt et al. (1980), Arkerlus and Srinivasan (1987), Datta and Paul (2001) used mark-up rate instead of price itself.

In addition to Padmanabhan and Vrat (1995), Wee (1999) and Wee and Law (2001) more recent literature on pricing decisions and lot sizing focused on deteriorating items. Teng and Chang (2005) established an EPQ model for deteriorating items considering that the demand rate depends on both selling price and on on-display stock, also imposing a ceiling on shelf size. You (2006) recently developed a perishable inventory model in which optimize price and maximize total profit. Dye et al. (2007) developed a deterministic EOQ model for deteriorating items with price-dependent demand is developed, also allowing shortage.

In the inventory literature, pricing and inventory joint decision-making has had significant attention. In this literature demand for a product has been assumed as function of its own selling price. But to our knowledge, there is no inventory-pricing model that explicitly includes the effects of complement products' selling price. This has been the case, since only the outsiders, instead of the product seller/producer, exogenously determine its complement's selling price. So, in general, a producer/seller does not have a direct power to affect its complements selling price. For this reason, in inventory models, the effects of complement good's price have been treated as embedded inside some part of the demand parameters, which also cover many other unmentioned effects as well. Even thought it is plausible to continue to think that a complementary good's selling price may be treated and implicitly assumed as part of constant term and other parameters, sometimes it would be very important to treat the complement's selling price in a separate setting so that it would be easier to understand its effects on the product's demand and optimal pricing behaviour. This argument would be even more significant as complement good's price considerably affects the product's selling price, demand and profit level.

In this paper, we analyze a production-inventory model that establish optimal selling price and lot size where demand rate is also affected by complement products' selling price.

4. THE MODEL

We assume that demand for the product is function of time, selling price, substitute (rival) product's selling price and compliment product's selling price. The study by Teng et al. (2005) entails that product life is significantly reduced in today's world and time is an important factor to incorporate into inventory modelling. Other examples on studies on the time-varying demand include Wagner and Whitin, (1958), Urban and Baker (1997), Balkhi and Benkherouf (2004) and You (2005). Time factor is also added as it is assumed that demand for the product grows over time. We use power form demand function.

$$D\left(t,P_k\right)=\alpha\, e^{aP_r-bP_c-cP_k+\beta t}$$

α,β,a,b,c are scale parameters, where $\alpha>0$ and $a,b,c,\beta\geq 0$

P_k Product's selling price in kth cycle

P_c Compliment product's selling price

P_r Rival product's selling price

C_u Unit cost

h Holding cost

C_o Order cost

Change in inventory level at time t is

$$\frac{dI_k(t)}{dt} = -D(t, P_k)$$

$$= -\alpha\, e^{\lambda - cP_k + \beta t} \tag{1}$$

where, $\lambda = aP_r - bP_c$ is assumed for simplicity.

Using initial condition, $I_k(T_{k-1}) = Q_k$, and terminal condition, $I_k(T_k) = 0$, solution of differential equation (1) is,

$$I_k(t) = A\left(e^{\beta T_k} - e^{\beta t}\right), \quad T_{k-1} \le t \le T_k \tag{2}$$

and

$$T_k = \frac{\ln\left(\frac{Q_k}{A} + e^{\beta T_{k-1}}\right)}{\beta}$$

where $A = (\alpha / \beta) e^{\lambda - cP_k}$

The incurred holding cost for the kth cycle, HC_k, is

$$HC_k = h \int_{T_{k-1}}^{T_k} I_k(t)\,dt$$

$$= hA\left[\left(T_k - T_{k-1} - \frac{1}{\beta}\right)\left(\frac{Q_k}{A} + e^{\beta T_{k-1}}\right) + \frac{e^{\beta T_{k-1}}}{\beta}\right]$$

The purchasing cost, PC_k, is

$$PC_k = C_u Q_k + C_o$$

Adding above cost components, the total cost for the kth cycle, TC_k, is achieved:

$$TC_k = PC_k + HC_k$$

Total revenue is also needed in order to obtain the total profit. Thus, the total revenue during the kth cycle, TR_k, is

$$TR_k = P_k Q_k$$

Using the total revenue and total cost functions, the average total profit per unit time for kth cycle, TPU_k, is attained:

$$TPU_k = \frac{TR_k - TC_k}{T_k - T_{k-1}}$$

5. NUMERICAL EXAMPLE

We present a numerical example where the parameter values are $a = 0.20$, $b = 0.22$, $c = 0.24$, $P_r = 21$, $P_c = 18$, $\alpha = 3000$, $\beta = 0.16$, $h = 2$, $C_u = 8$, $C_o = 100$. Our aim it to obtain optimal values of selling price as well as order quantities in kth cycle. For the first six cycles the solution results of the model are presented in Table 1. Findings indicate that there is decrease in optimal selling prices but increase in order quantities as moved from one cycle to another. In parallel per unit profit increase dramatically yet the cycle time shortens.

Table 1: Optimal Solution Results

k	$T_k - T_{k-1}$ kth cycle lenght	TPU_k	Q_k	P_k
1	0.8931	627.80	158.6	13.08
2	0.8281	742.08	171.6	13.01
3	0.7736	864.90	184.4	12.96
4	0.7254	996.45	197.3	12.91
5	0.6824	1136.57	210.2	12.86
6	0.6448	1285.07	223.0	12.82
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6. SENSITIVITY ANALYSIS FOR THE DEMAND PARAMETERS

Sensitivity analyses for the demand parameters have been performed. The results are presented in Table 2. Effect of higher values of α on optimal values of price and total profit per unit time is the first in the Table. For each cycle of k higher values of α reduce the optimal selling prices but significantly increase the total profit per unit time.

Table 2: Response of Optimal Solutions to Changes in Demand Parameters

k	$\alpha = 2500$		$\alpha = 3500$		$\alpha = 4500$	
	P_k	TPU_k	P_k	TPU_k	P_k	TPU_k
1 2 3 . .	13.18 13.09 13.02	505.39 609.61 722.37	13.01 12.95 12.90	751.85 875.48 1007.65	12.90 12.86 12.82	1003.63 1144.03 1293.00
	$\beta = 0.14$		$\beta = 0.18$		$\beta = 0.22$	
1 2 3 . .	13.06 13.00 12.95	621.60 717.82 820.24	13.11 13.03 12.96	634.19 768.19 913.81	13.17 13.06 12.97	647.61 826.73 1025.54
	$a = 0.18$		$a = .22$		$a = 0.26$	
1 2 3 . .	13.31 13.21 13.12	378.31 470.48 571.52	12.90 12.85 12.82	1020.35 1161.95 1311.87	12.64 12.62 12.60	2586.50 2803.37 3028.51
	$b = 0.20$		$b = 0.24$		$b = 0.28$	

1 2 3 . .	12.92 12.88 12.83	952.94 1090.16 1236.08	13.28 13.18 13.10	407.34 502.51 606.40	13.83 13.60 13.44	159.76 225.69 300.67
	$c = 0.22$		$c = 0.26$		$c = 0.30$	
1 2 3 . .	13.40 13.34 13.29	853.11 992.58 1141.69	12.83 12.75 12.69	459.94 554.50 656.86	12.51 12.39 12.30	239.47 305.78 378.67

Upward changes in the parameter of time, β, seem to increase price levels as well as total profit per unit time. Increases in price elasticity of substitution, a, suggests higher levels of sensitivity in of selling price. Thus higher levels of a causes lower optimal price levels. However this significantly increases the total profit per unit time. Profitability continues to increase in upcoming cycles. Sensitivity with respect to changes in complementary product's selling prices is considered with the parameter b. Higher levels of b increase the optimal selling prices while significantly reduce profitability. Lastly effect of elasticity of demand, c, on optimal values of selling price and profitability is considered. For each cycle, higher levels of c increases the sensitivity of customers to price levels, which convey to lower levels of selling price and profitability.

7. CONCLUSION

Product demand is sensitive to changes in selling price. But in real life there are other parameters that affect the demand level including, time, complementary and substitute product's selling prices. This effect can be observed practically in many sold items all over the markets. Thus, in this paper, we develop an inventory model where the demand level is sensitive to changes in parameters of its own and its complementary as well as substitute good's selling prices. After the model development, the situation has been illustrated with a numerical example. Optimal solution results and sensitivity analysis on demand parameters have been performed.

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